A Nonresonant Perturbation Theory

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Abstract—This paper presents a theory for a nonresonant perturbation technique for the measurement of electric and magnetic field strengths within a device. Most presently employed perturbation field strength measurements require the use of a resonance technique. In the technique discussed here, reflection coefficient measurements are made at the same frequency with, and without, a perturbing object placed at the point at which the field strength is to be measured. By these data, and by the equations derived and presented in this paper, the desired field strength can be calculated.

The technique can be used for cavities that are too lossy to support resonance, and is suitable for cavities for which the resonant field configuration differs from the field configuration to be measured. In addition, this technique has the advantage that it permits the measurement of the phase, as well as the amplitude of the field.

INTRODUCTION

PERTURBATION techniques have been used for decades for the measurement of electromagnetic waves within devices. As early as 1937 Harries [1] found the electric field direction by a resonance perturbation technique. In 1952 Maier and Slater [2] presented the well-used resonance perturbation method for measurement of field strength. By this technique, the frequency perturbation of a resonator by a dielectric or conducting object is used to obtain the field.

In some cases, however, electric field measurements are desired in devices in which resonance cannot be employed. The device may be too lossy to support resonance. Alternatively, one may wish to know the field strength in the device under nonresonant conditions of operation; the corresponding field strengths in the device when in resonance may be considerably different.

For these reasons, a number of papers on nonresonant perturbation techniques have been presented and employed within the last few years. The nonresonant techniques are characterized by the fact that the frequency at which the measurements are made remains fixed. That is, this frequency is independent of movement of the perturbing object within the device, as well as its removal from the device. Generally, the nonresonant techniques fall into two categories: 1) those in which the reflection coefficient at an input port is measured, and 2) those in which the device is treated as a two-port, for which the transmission coefficient is measured.

In the 1955 Kino [3] presented a theory for a technique of the latter type. This theory was for use in uniform and periodically-loaded waveguides. Specifically, he calculated the change in propagation constant of the device in terms of the field strength and the parameters of the perturbing object. In 1957 Lagerstrom [4] presented a similar theory, again for transmission coefficient measurement, as well as the results of measurements he had performed on periodically-loaded waveguides.

In the past three years, considerable interest and activity have been devoted to nonresonant perturbation measurements in which the input reflection coefficient of the device is measured. Measurements have been made not only on periodically-loaded transmission lines but also on devices having a wide variety of sizes and shapes. In 1958, at the Stanford University Microwave Laboratory, K. Mallory and R. Miller measured the electric field on-axis in a periodically-loaded waveguide in this manner. More recently this technique has been used by R. Borghi to measure the electric field on-axis in sections of accelerator waveguide, and in a variety of other components for the Stanford two-mile linear accelerator. Mallory [5] presented a nonresonant technique of this type in 1961. This paper supplements the Mallory presentation by providing a more rigorous justification for the measurement technique. The principal value of this paper is that it provides a rigorous, general theory for measurements of this type.

This paper provides a theory for a nonresonant perturbation technique for measuring the electric and magnetic fields at various points within a device. It is adapted specifically to a steady-state field with a sinusoidal time variation. Its value will be found primarily in microwave devices. The device can be a transmission line, waveguide, or, in fact, any object that has the following properties.

1) Basically, the device consists of a cavity that contains an electromagnetic field. (See Fig. 1.)

2) Electromagnetic power is permitted to enter the cavity only at a single port while perturbation measurements are being made. (This is the port at which reflection coefficient measurements are made.)

3) A single TE, TM, or TEM mode is present in that part of the input waveguide where reflection coefficient measurements are made.

4) The walls of the cavity provide a very high degree of isolation between its interior and its exterior for electromagnetic waves at the operating frequency. (In most practical cases these walls are made of highly conducting metals.)

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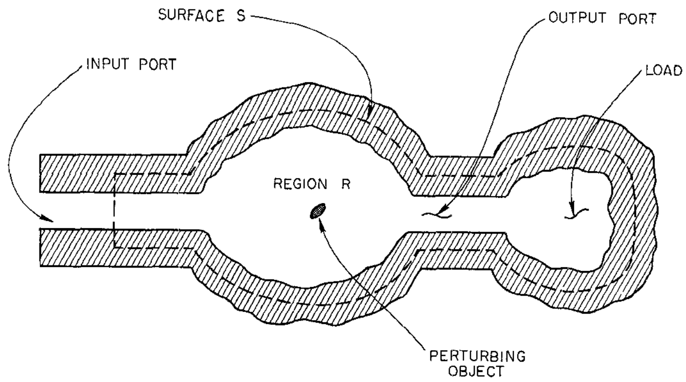

Fig. 1. Cavity in which fields are to be measured.

5) The cavity walls and the medium inside the cavity are assumed to have electrical parameters that are linear and isotropic.

Basic Theory

Figure 1 shows a cross-sectional view of the cavity. It has just one waveguide (or transmission line) port through which electromagnetic energy is permitted to pass into its interior. It can have any size or shape. The cavity can be either lossy (in its walls, its interior, or both) or lossless.

The cavity, as defined in the introduction and in the theoretical development to be given, is considered to include any output waveguides that the device might have, as well as the loads to which they connect. This concept is illustrated with one output waveguide and load in Fig. 1.

Consider now the region R, of volume V, inside the closed surface S in Fig. 1. As shown, the surface S lies entirely within the cavity walls, except where it crosses the input waveguide in a plane normal to the waveguide axis.

The basic formulation for this theory is similar to the Lorentz Reciprocity Theorem [6] and to the theory developed by Jaynes [7] to calculate the change in the input impedance of a cavity when it is modified. Two different electromagnetic fields are considered within region R. One field, in the absence of a perturbing object, is designated by the electric and magnetic field components $\boldsymbol{E}_a$ and $\boldsymbol{H}_a$, respectively. The other field, in the presence of a perturbing object within region R, is designated by the electric and magnetic field components $\boldsymbol{E}_p$ and $\boldsymbol{H}_p$, respectively. These two fields have the same frequency. We employ the vector $\boldsymbol{p}$ defined by

$$\boldsymbol{p} = \boldsymbol{E}_a \times \boldsymbol{H}_p - \boldsymbol{E}_p \times \boldsymbol{H}_a \tag{1}$$

throughout region R and over the surface S. The first step in the derivation is to relate $\boldsymbol{p}$ over the surface S to $\boldsymbol{p}$ throughout volume V by the divergence theorem [8],

$$\int_S (\boldsymbol{n}\cdot\boldsymbol{p})ds = \int_V (\nabla\cdot\boldsymbol{p})dv \tag{2}$$

where $\boldsymbol{n}$ is the unit vector, normally outward from surface S. In (2) the integral on the left is over the entire closed surface S, and the integral on the right is throughout all of the volume V, contained in region R. In the paragraphs to follow, the integrals in (2) are developed into forms suitable for use in perturbation measurement.

Consider first the integral on the left of (2). Suppose that surface S consists entirely of two parts: S_1, the part that crosses the waveguide input port; and S_2, the part contained within the cavity wall. We assume that the cavity wall attenuates electromagnetic waves so effectively that, over surface S_2 (which lies between the inner and outer cavity walls),

$$\boldsymbol{E}_a = \boldsymbol{H}_a = \boldsymbol{E}_p = \boldsymbol{H}_p = \boldsymbol{p} = 0.$$

Thus

$$\int_S (\boldsymbol{n}\cdot\boldsymbol{p})ds = \int_{S_1} (\boldsymbol{n}\cdot\boldsymbol{p})ds. \tag{3}$$

Now over surface S_1, using (1),

$$\begin{aligned} \boldsymbol{n}\cdot\boldsymbol{p} &= \boldsymbol{n}\cdot(\boldsymbol{E}_a \times \boldsymbol{H}_p) - \boldsymbol{n}\cdot(\boldsymbol{E}_p \times \boldsymbol{H}_a) \\ \boldsymbol{n}\cdot\boldsymbol{p} &= (\boldsymbol{n} \times \boldsymbol{E}_a)\cdot\boldsymbol{H}_p - (\boldsymbol{n} \times \boldsymbol{E}_p)\cdot\boldsymbol{H}_a \\ \boldsymbol{n}\cdot\boldsymbol{p} &= (\boldsymbol{n} \times \boldsymbol{E}_{as})\cdot\boldsymbol{H}_{ps} - (\boldsymbol{n} \times \boldsymbol{E}_{ps})\cdot\boldsymbol{H}_{as}. \end{aligned} \tag{4}$$

In (4), the subscript s denotes those components of the fields that lie in the plane surface S_1. Suppose now that over S_1, $\boldsymbol{E}_a$ and $\boldsymbol{H}_a$ are composed entirely of a single waveguide mode, and that $\boldsymbol{E}_p$ and $\boldsymbol{H}_p$ are composed entirely of the same waveguide mode. At each point on S_1, then, $\boldsymbol{E}_{as}$ and $\boldsymbol{E}_{ps}$ must lie in the same direction, and $\boldsymbol{H}_{as}$ and $\boldsymbol{H}_{ps}$ must lie in the same direction. In a single waveguide mode, the components of $\boldsymbol{E}$ and $\boldsymbol{H}$ that lie in a cross-sectional plane must be perpendicular to each other. In (4), the vectors $(\boldsymbol{n}\times\boldsymbol{E}_{as})$ and $(\boldsymbol{n}\times\boldsymbol{E}_{ps})$ are perpendicular to $\boldsymbol{E}_{as}$ and $\boldsymbol{E}_{ps}$, but are parallel to $\boldsymbol{H}_{as}$ and $\boldsymbol{H}_{ps}$. Thus, (4) becomes

$$\boldsymbol{n}\cdot\boldsymbol{p} = E_{ps}H_{as} - E_{as}H_{ps} \tag{5}$$

where E_{as}, H_{as}, E_{ps}, and H_{ps} are all scalars. In general, these fields contain incident and reflected waves within the waveguide, and can be expressed as

$$E_{as} = (1 + \Gamma_a)E_{asi} \tag{6}$$

$$H_{as} = (1 - \Gamma_a)H_{asi} \tag{7}$$

$$E_{ps} = (1 + \Gamma_p)E_{psi} \tag{8}$$

$$H_{ps} = (1 - \Gamma_p)H_{psi} \tag{9}$$

where Γ_a and Γ_p are the reflection coefficients at S_1 (the input port) in the absence of the perturbing object and in its presence, respectively. In these equations, the subscript i denotes the incident wave. When (5) is combined with (6) through (9), and one notes that

$$\frac{E_{psi}}{H_{psi}} = \frac{E_{asi}}{H_{asi}}$$

the result is

$$\boldsymbol{n}\cdot\boldsymbol{p} = (\Gamma_p - \Gamma_a)(E_{asi}H_{psi} + E_{psi}H_{asi}). \quad (10)$$

The field components in (10) are all taken to have phase angles of zero over the reference plane S_1. This causes no loss in generality.

From Poynting's Theorem [9], and the fact that the E- and H-field components in (10) are perpendicular to each other, it is apparent that

$$\int_{S_1} (E_{asi}H_{psi} + E_{psi}H_{asi})\, ds = 2\sqrt{P_{ai}P_{pi}}. \quad (11)$$

In (11) P_{ai} and P_{pi} are the power levels in the incident waves that pass through S_1, in the absence of, and in the presence of, the perturbing object, respectively. When (3), (10), and (11) are combined, the result is

$$\int_S (\boldsymbol{n}\cdot\boldsymbol{p})\, ds = 2\sqrt{P_{ai}P_{pi}}\,(\Gamma_p - \Gamma_a),$$

and since, in common practice, the incident wave power levels are equal in the presence and absence of the perturbing object, $P_{ai}=P_{pi}=P_i$ and

$$\int_S (\boldsymbol{n}\cdot\boldsymbol{p}) ds = 2P_i(\Gamma_p - \Gamma_a). \quad (12)$$

Consider now the term on the right of (2). From (1)

$$\nabla\cdot\boldsymbol{p} = \nabla\cdot(\boldsymbol{E}_a \times \boldsymbol{H}_p) - \nabla\cdot(\boldsymbol{E}_p \times \boldsymbol{H}_a),$$

which by means of a vector identity becomes

$$\nabla\cdot\boldsymbol{p} = (\nabla \times \boldsymbol{E}_a)\cdot\boldsymbol{H}_p - (\nabla \times \boldsymbol{H}_p)\cdot\boldsymbol{E}_a - (\nabla \times \boldsymbol{E}_p)\cdot\boldsymbol{H}_a + (\nabla \times \boldsymbol{H}_a)\cdot\boldsymbol{E}_p. \quad (13)$$

When Maxwell's Equations, given by

$$\nabla \times \boldsymbol{E} = -j\omega\mu\boldsymbol{H}$$

and

$$\nabla \times \boldsymbol{H} = \boldsymbol{i}_c + j\omega\epsilon\boldsymbol{E} = \boldsymbol{i}_c + \boldsymbol{i}_d = \boldsymbol{i}_t$$

are substituted into (13), it becomes

$$\nabla\cdot\boldsymbol{p} = -j\omega(\mu_a - \mu_p)\boldsymbol{H}_a\cdot\boldsymbol{H}_p + \boldsymbol{E}_p\cdot\boldsymbol{i}_{ta} - \boldsymbol{E}_a\cdot\boldsymbol{i}_{tp} \quad (14)$$

where $\boldsymbol{i}_s$, $\boldsymbol{i}_d$, and $\boldsymbol{i}_t$ are the conduction, displacement, and total current densities, respectively.

By the Lorentz Reciprocity Theorem one can see that in the region R outside the perturbing object

$$\nabla\cdot\boldsymbol{p} = 0,$$

since at every such point the conductivity, permittivity, and permeability are the same with and without the perturbing object. As a result

$$\int_V (\nabla\cdot\boldsymbol{p}) dv = \int_{V_p} (\nabla\cdot\boldsymbol{p}) dv \quad (15)$$

where V is the volume throughout region R, and V_p is only the volume occupied by the perturbing object.

When (2), (12), (14), and (15) are combined, the result is

$$2P_i(\Gamma_p - \Gamma_a) = \int_{V_p} (\boldsymbol{E}_p\cdot\boldsymbol{i}_{ta} - \boldsymbol{E}_a\cdot\boldsymbol{i}_{tp} - j\omega(\mu_a - \mu_p)\boldsymbol{H}_a\cdot\boldsymbol{H}_p) dv. \quad (16)$$

Expression in Terms of Electric and Magnetic Dipole Moments

If the perturbing object is quite small compared to a wavelength, its scattered field consists entirely of the radiation from an electric dipole moment and a magnetic dipole moment. For such an object, the right side of (16) can be replaced by an expression in terms of these dipole moments, as shown in the following.

The first step in this derivation is to show that the input reflection coefficient change caused by the perturbing object depends upon the electric and magnetic dipole moments that it sets up, but is otherwise completely independent of its properties. Combining (3), (5), and (12) yields

$$2P_i(\Gamma_p - \Gamma_a) = \int_{S_1} (E_{ps}H_{as} - E_{as}H_{ps})\, ds. \quad (17)$$

Now, let $E_{\Delta s}$ and $H_{\Delta s}$ be the components of the electric and magnetic fields scattered by the perturbing object, that lie in the input plane S_1 that crosses the input waveguide. Then

$$E_{ps} = E_{\Delta s} + E_{as}$$
$$H_{ps} = H_{\Delta s} + H_{as},$$

and when these equations are substituted into (17) the result is

$$2P_i(\Gamma_p - \Gamma_a) = \int_{S_1} (E_{\Delta s}H_{as} - E_{as}H_{\Delta s})\, ds. \quad (18)$$

In turn, one can express $E_{\Delta s}$ and $H_{\Delta s}$ in terms of the electric and magnetic dipole moments set up by the perturbing object, $\boldsymbol{P}$ and $\boldsymbol{M}$, by

$$E_{\Delta s} = \boldsymbol{C}_1\cdot\boldsymbol{P} + \boldsymbol{C}_2\cdot\boldsymbol{M} \quad (19)$$

$$H_{\Delta s} = \boldsymbol{C}_3\cdot\boldsymbol{P} + \boldsymbol{C}_4\cdot\boldsymbol{M}. \quad (20)$$

In (19) and (20) the vectors $\boldsymbol{C}_1$, $\boldsymbol{C}_2$, $\boldsymbol{C}_3$, and $\boldsymbol{C}_4$ represent the coupling between the dipole moments and the field components in the plane S_1 (see Fig. 1).

When (18), (19), and (20) are combined, the result is

$$2P_i(\Gamma_p - \Gamma_a) = \boldsymbol{k}_1\cdot\boldsymbol{P} + \boldsymbol{k}_2\cdot\boldsymbol{M} \quad (21)$$

where

$$\boldsymbol{k}_1 = \int_S (H_{as}\boldsymbol{C}_1 - E_{as}\boldsymbol{C}_3)\, ds \quad (22)$$

and

$$k_2 = \int_S (H_{as}C_2 - E_{as}C_4)\, ds. \quad (23)$$

Equations (22) and (23) show that k_1 and k_2 are completely independent of the perturbing object. It is apparent then from (21) that the reflection coefficient change depends upon the properties (size, shape, composition, etc.) of the perturbing object only to the extent that they affect its electric and magnetic dipole moments.

To evaluate k_1, we chose a perturbing object that consists of two identical balls, separated from each other and connected by a very thin wire, all perfectly conducting. The separation between the balls is quite large compared to their radii. Since the device is perfectly conducting, the electric and magnetic fields in its presence, $\boldsymbol{E}_p$ and $\boldsymbol{H}_p$, are zero inside it, with the result that (16) becomes

$$2P_i(\Gamma_p - \Gamma_a) = -\int_{V_p} (\boldsymbol{E}_a \cdot \boldsymbol{i}_{tp})\, dv. \quad (24)$$

Since $\boldsymbol{E}_a$ is considered uniform throughout the space to be occupied by the perturbing object, (24) becomes

$$2P_i(\Gamma_p - \Gamma_a) = -\boldsymbol{E}_a \cdot \int_{V_p} \boldsymbol{i}_{tp}\, dv = -\boldsymbol{E}_a \cdot (I_{tp}\boldsymbol{l}) \quad (25)$$

where I_{tp} is the total current that flows along the wire, and $\boldsymbol{l}$ is a vector whose direction is that of the perturbing object and whose magnitude is its length. If Q is the charge on one of the balls, then

$$I_{tp}\boldsymbol{l} = j\omega Q\boldsymbol{l}$$

and since

$$\boldsymbol{P} = Q\boldsymbol{l},$$

then

$$I_{tp}\boldsymbol{l} = j\omega \boldsymbol{P}. \quad (26)$$

Combining (25) and (26) yields

$$2P_i(\Gamma_p - \Gamma_a) = -\boldsymbol{E}_a \cdot (j\omega \boldsymbol{P}). \quad (27)$$

Since this dipole, acting under the electric field, produces zero magnetic moment, it is evident by comparing (21) and (27) that

$$k_1 = -j\omega \boldsymbol{E}_a. \quad (28)$$

To evaluate k_2, we chose a perturbing object that consists of a circular wire ring, again perfectly conducting, with the result that (24) again applies. By taking the total current in the ring to be I_{tp}, and assuming it to be constant around the ring, (24) becomes

$$2P_i(\Gamma_p - \Gamma_a) = -I_{tp}\oint \boldsymbol{E}_a \cdot d\boldsymbol{l}. \quad (29)$$

The magnetic flux Φ that threads the loop is given by

$$\Phi = A\mu_a \boldsymbol{H}_a \cdot \boldsymbol{n}$$

where $\boldsymbol{n}$ is a unit vector normal to the plane of the loop, and A is its enclosed area. Therefore,

$$\oint \boldsymbol{E}_a \cdot d\boldsymbol{l} = -j\omega\Phi = -j\omega\mu_a A \boldsymbol{H}_a \cdot \boldsymbol{n}. \quad (30)$$

By combining (29) and (30), one obtains

$$2P_i(\Gamma_p - \Gamma_a) = j\omega\mu_a I_{tp} A \boldsymbol{H}_a \cdot \boldsymbol{n}$$

and since the magnetic dipole moment $\boldsymbol{M}$ is given by

$$\boldsymbol{M} = I_{tp} A \boldsymbol{n},$$

then

$$2P_i(\Gamma_p - \Gamma_a) = j\omega\mu_a \boldsymbol{M} \cdot \boldsymbol{H}_a. \quad (31)$$

Comparison of (21) and (31) shows that

$$k_2 = j\omega\mu_a \boldsymbol{H}_a. \quad (32)$$

The values of k_1 and k_2 shown in (28) and (32) are completely independent of the perturbing object. When these values are substituted into (21) the result is

$$2P_i(\Gamma_p - \Gamma_a) = -j\omega[\boldsymbol{E}_a \cdot \boldsymbol{P} - \mu_a \boldsymbol{H}_a \cdot \boldsymbol{M}]. \quad (33)$$

Expression in Terms of Polarizability

The concept of polarizability [10] can be applied to a certain class of perturbing objects. Such objects have the property that if one is placed in a sinusoidally varying electric field, it sets up an electric dipole moment, but no magnetic dipole moment. Conversely, if it is placed in a sinusoidally varying magnetic field, it sets up a magnetic dipole moment, but no electric dipole moment. Perturbing objects used in practical field strength measurements usually have this property. There are two advantages to the use of the polarizability concept in connection with perturbation field strength measurements. First, the formulation (to be developed later) is more readily useable than either (16) or (33). Second, it permits the use of existing formulas for the polarizabilities of variety of perturbing objects of different shapes [11].

The electric and magnetic dipole moments can be expressed by

$$\boldsymbol{P} = \epsilon[\alpha_e] \cdot \boldsymbol{E}_a$$
$$\boldsymbol{M} = [\alpha_m] \cdot \boldsymbol{H}_a$$

where α_3 and α_m are tensor polarizabilities. When these equations are substituted into (33), the result is

$$2P_i(\Gamma_p - \Gamma_a) = -j\omega[\epsilon_a(\boldsymbol{E}_a \cdot [\alpha_e]) \cdot \boldsymbol{E}_a - (\mu_a \boldsymbol{H}_a \cdot [\alpha_m]) \cdot \boldsymbol{H}_a]. \quad (34)$$

In practice, it is much easier to use scalar polarizabilities than tensor polarizabilities. This can be done with a class of perturbing objects having an additional restriction. These are objects that have rotational symmetry about an axis, symmetry about a plane normal to the axis, and electric and magnetic polarizabilities that are scalar in the direction of the axis, and in the direction normal to the axis. A polarizability is scalar if the electric or magnetic field causes a corresponding dipole moment that is collinear to the field. For such a perturbing object one can show easily that (34) reduces to

$$2P_i(\Gamma_p - \Gamma_a) = -j\omega[\epsilon\alpha_e E_a^2 - \mu\alpha_m H_a^2] \tag{35}$$

where

$$\alpha_e = \alpha_{ep}\cos^2\theta_e + \alpha_{en}\sin^2\theta_e \tag{36}$$

$$\alpha_m = \alpha_{mp}\cos^2\theta_m + \alpha_{mn}\sin^2\theta_m. \tag{37}$$

In (36) and (37), θ_e and θ_m are the angles between the axis of the perturbing object and the impressed electric and magnetic fields, respectively. The terms α_{ep}, α_{en}, α_{mp}, and α_{mn} are the scalar polarizabilities, with α_{ep} and α_{mp} taken parallel to the axis of the perturbing object, and α_{mp} and α_{mn} taken normal to that axis.

Comparison with Slater Resonant Theory

Maier and Slater [2] derived formulas that show resonant frequency change resulting from the use of conducting oblate and prolate spheroids as perturbing objects. These formulas show this frequency change as a function of the size and shape of each of these objects for electric and magnetic fields parallel to and normal to its axis. Maier and Slater also provide a set of curves calculated from these formulas. Each curve shows how the frequency change varies with shape of the spheroid, when it perturbs either the electric or magnetic field either parallel to, or normal to, the spheroid axis. Ginzton [12] repeats both the formulas and curves of Maier and Slater.

Using the formulas presented previously, one can calculate the change in reflection coefficient that results when a conducting oblate or prolate spheroid is used as a perturbing object in the nonresonant technique. For any combination of electric or magnetic fields, at the point of perturbation, one can make this calculation in either of two ways. First, one can calculate directly using (16). The other, and much easier way, is to use (35), (36), (37) and for the polarizabilities α_{ep}, α_{en}, α_{mp}, and α_{mn} to use the polarizability formulas given by Collin [11]. Either way one finds that the quantity $(\Gamma_p - \Gamma_a)$ for the nonresonant technique varies linearly with the quantity

$$\frac{\omega_0^2 - \omega^2}{\omega_0^2}$$

for the Slater resonant technique, with changes in the size and shape of the perturbing object, and changes in the directions and magnitudes of the electric and magnetic fields at the point of perturbation. This, of course, is to be expected. As a result, the Maier and Slater curves for conducting oblate and prolate spheroids apply equally well to the nonresonant perturbation technique.

Note: The experimental applications of this author's theory are discussed in the Correspondence section of this issue by K. B. Mallory and R. H. Miller, "On Nonresonant Perturbation Measurements."

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